The use of technology provides an effective way for promoting multiple representations in problem solving and mathematics. Multiple representations allow students to experience different ways of thinking, develop better insights and understandings of problem situations, and increase comprehension about mathematical concepts. Even with all the benefits of multiple representations, however, teachers find it difficult to incorporate open-ended problem solving that capitalizes on these representations because of time constraints and limitations of traditional mathematics teaching. Technology can become a vital and exciting tool in allowing students to explore multiple representations...
representations and mathematical situations and relationships (NCTM 2000). Technology empowers students who may have limited mathematical knowledge and limited symbolic and numeric manipulation skills to investigate problem situations. Technology not only frees the students from tedious and repetitive computations but actually encourages the use of multiple representations. Students can easily move from a spreadsheet to a graph or geometry software in their quest for solutions to a given problem. When supported by the teacher, these tools of technology provide students with opportunities to investigate and manipulate mathematical situations to observe, experiment with, and make conjectures about patterns, relationships, tendencies, and generalizations. Teachers should emphasize and encourage the use of multiple representations to support students’ thinking and understanding of concepts and problem-solving situations in all areas of mathematics.

In this article, we will elaborate on NCTM’s vision for technology use by highlighting the flexibility and richness that it offers in problem solving through a multiple-representation approach. “When technological tools are available, students can focus on decision making, reflection, reasoning, and problem solving” (NCTM 2000, p. 25). We offer such use of technology with two problems adapted from the InterMath Web site (www.intermath-uga.gatech.edu/). InterMath is designed to support teachers in developing mathematical understanding. Some investigations on the Web site are appropriate for middle school classrooms, and teachers involved in InterMath have adapted other investigations to use with their students. The intent of InterMath problems, whether for students or teachers, is to support the development of understanding, using, and appreciating mathematics. Consistent with the multiple-representations theme of this article, InterMath problems encourage learners to work with the problems in multiple ways. The vignettes that follow offer ideas for supporting students in working with open-ended problems that lend themselves to multiple representations and help provide concrete examples of what technology-enhanced mathematics can look like.

Jack’s Mealworms

Jack and Jill each have two mealworms. Jack’s mealworm population will triple each month. Jill’s mealworm population will increase by 40 each month. During what month(s) will Jack and Jill once again have the same number of mealworms? How many mealworms will each of them have?

This kind of problem promotes the use of problem-solving strategies through the investigation of mathematical concepts. In this problem, students are required to extend a pattern by using the first term and the rule of the pattern. Although this problem could be solved using pencil and paper to complete a table, it provides an excellent opportunity for technology to be integrated into instruction. By using graphing software or a spreadsheet, students will quickly be able to represent the problem in multiple ways and investigate whether there are multiple solutions to the problem.

A spreadsheet provides one effective way to model this problem, since it allows a pattern to be extended quickly and easily. In a spreadsheet, students can create their table by using three columns. The first column is the month, which increases by 1. Let the next column represent Jack’s mealworms, which start at 2 and triple each month. The final column represents Jill’s mealworms, which start at 2 and increase by 40 each month. The number of mealworms that Jack has will always be the number of mealworms from the previous month times 3. The number of mealworms that Jill has will always be the number of mealworms from the previous month plus 40.

As shown in figure 1, Jill has more mealworms than Jack until the end of the fourth month, when they each have 162 mealworms. After the fourth month, Jack has more mealworms than Jill. The use of a spreadsheet allows students to extend the pattern without tedious calculations that may prohibit them from looking for multiple solutions or other emerging patterns. For example, in this problem, when the pattern is extended for thirty months, for example, Jack’s mealworm population is $2.1 \times 10^{14}$, whereas Jill’s population is 1202. Students can easily see that Jack’s number is increasing more rapidly than Jill’s. By analyzing the pattern in the table, students should be able to tell that Jill will never again have as many worms as Jack.

This problem features two patterns—exponential and linear. Once the patterns have been identified and discussed, the spreadsheet allows us to ex-
tend the pattern quickly. As with any open-ended investigation, technology-enhanced investigations require the teacher to take an active role in supporting students to find generalizations and make sense of mathematical ideas. In this situation, the teacher could lead a discussion regarding the two patterns, helping students see that Jack’s pattern is exponential and that Jill’s pattern is linear.

The equation for Jack’s mealworms is exponential, since it triples each month. This problem leads to a discussion about exponential equations, which can be facilitated by the question, “How are exponential and linear patterns different?” Student responses may include the following kinds of observations:

- The previous term is being multiplied by 3 to calculate the next term.
- A linear pattern involves adding a constant amount each term, which is not occurring here.
- The terms in the pattern are growing large quickly, which indicates an exponential pattern.

To capitalize on helping students understand exponential patterns, the teacher may want to ask them to consider what information repeats in the pattern. Looking at figure 2, students should notice that the number of mealworms for a given month equals the previous month’s number times 3. To reinforce the move to symbolic manipulation, you may want to work with your students to explore how exponents can be used to simplify the pattern, as in figure 2. The equation for Jack’s mealworm population, if \( x \) = months, is \( 2(3^x) \). Jill’s mealworm population, however, can be modeled with a linear equation, since it increases by the same number each month. Jill started with two mealworms, and each month her population increases by 40. The pattern is also extended in figure 2. The graph can be put into slope-intercept form, a topic that most middle school students would have experienced. If \( x \) = months, the number of Jill’s mealworms equals 40\( x \) + 2.

Once the students have identified the equations and nature of the equations, the class discussion should support the students in developing generalizations from the table. For example, after looking at the extended pattern on the spreadsheet, students should form the generalization that other than at the beginning, Jack and Jill will only have one month when they have the same number of mealworms.

Of course, simply stopping with the spreadsheet solution leaves students with an incomplete understanding of the problem. The question can be answered, or more fully understood, by using graphing software or a graphing calculator. By graphing the equations for the patterns, as shown in figures 3 and 4, linear and exponential functions can be explored. To guide the use of the graph of the two equations, the teacher can ask the students to consider the following question: “If the number of mealworms continues to grow at the same rate, will Jack and Jill ever have the same number of mealworms again?”

Students can derive their own equations or if the class has developed an equation, it can be entered into graphing software. The graphs allow students a different way to develop an understanding of the difference between exponential and linear equations. At this point, ask the students to again explore how linear and exponential patterns are different. You may also work with the students to develop an understanding of what it means when the lines of the graphs cross each other. This activity becomes a powerful way for students to understand intersection, given that they will see it in both numerical and graphical forms.
The mealworm problem focuses on reinforcing mathematical knowledge and skills, as it requires students to identify the rule of a pattern, extend patterns, and write equations for functions. The use of technology enriches learning by providing multiple representations in the forms of spreadsheets and graphs. Students can see the problem graphically, numerically, and symbolically. The integration of technology also allows students to have the freedom to focus on problem solving and interpretation rather than rote arithmetic if using paper and pencil.

**Polygon Angles**

What is the sum of the angles of a triangle? Of a quadrilateral? Of a pentagon? Of a hexagon? What is the sum of the angles in any convex polygon in terms of the number of sides?

The problem posed here allows students to investigate and develop the formula for finding the sum of the angles of polygons. Approaching this problem through investigation gives students the chance to better understand it instead of focusing on computational skills to get an answer. There are many different approaches that students could take to solve this problem, ranging from paper-and-pencil methods to using calculators, spreadsheets, graphing calculators, or such geometry software as The Geometer’s Sketchpad (Sketchpad). The use of technology enables students multiple ways to generate solutions, form conjectures, and test those conjectures as they investigate the problem in an interactive learning environment.

With this problem, a good starting point would be to look at triangles using Sketchpad. The teacher can ask the students to create a wide variety of triangles and use the “measure angle” function in Sketchpad to find the sum of the angles (see Fig. 5). After working through a few examples, the teacher can ask each student to make a generalization about the total number of degrees in a triangle. Teachers can also encourage students to copy the angles of the triangle to form a straight line to see that the sum of the angles is always 180 degrees, information that becomes useful in finding the sums of the angles of other polygons. Sketchpad, at this point, is valuable because it allows students to quickly create, measure, and record the sums of angles in multiple shapes.

Before the students move on to more complex shapes, the teacher may want to go over class findings so far. It is likely that the students will have spent one class period on this task and will need to discuss their findings to make sense of them. Typically, student findings will be within about 1 degree of the expected 180 degrees because of rounding within the software. At this point, it will be important to help students understand why the measurement is not exactly “correct” and to develop a consensus among your students to call the sum of angles 180 degrees.
As a next step in solving this problem, ask students to repeat their exploration using quadrilaterals. As the students work with more complex shapes, the teacher may prompt students to employ the previous technique to find this sum (see fig. 6). To further student understanding, you may also ask students to look for a relationship between the minimum number of triangles needed to form the quadrilateral and the measure of the angles in the quadrilateral. Once students conclude that two triangles form a quadrilateral, they may feel confident in generalizing that the angles of a quadrilateral sum to $2 \times 180^\circ = 360^\circ$. If the teacher is comfortable with the progress being made using Sketchpad, the students may further delve into the study of the pentagon and then the hexagon.

Once the students have explored a variety of shapes with Sketchpad, you may ask them to move to a spreadsheet and incorporate these headings: type of polygon, number of sides, number of triangles, and sum of the angles. The students can use the “fill down” command to find the values for all $n$-gons (see fig. 7). In so doing, students can make generalizations about all $n$-gons, when $n$ is the number of sides of the polygon. You may prompt your students to look at the values in the columns involving the number of sides and the number of triangles. Students should see the pattern—the number of triangles that make up a polygon is 2 less than the number of sides. The formula $(n - 2) \times 180^\circ$ turns the sum of the angles of any polygon when the number of sides is known.

To tie this investigation to other mathematical problems, you may also want your students to develop a working knowledge of the symbolic manipulation that will yield the answers. One way of solving this problem symbolically would be by using a graphing calculator, graphing calculator software, or the “plot function” within the spreadsheet to plot some of the data points found. Students may plot the sum of the angles as it compares with the number of sides of the polygon (see fig. 8). The graph shows that the data points are linear, which may lead to a discussion on finding the equation that is represented. Finding the equation simply involves using any two points to find the slope, then calculating the $y$-intercept. For the slope-intercept form of a linear equation, $y = mx + b$. See the following example.

Using points $(3, 180)$ and $(4, 360)$:

$$slope = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{360 - 180}{4 - 3} = \frac{180}{1} = 180$$

Since $y = mx + b$,

then

$$360 = 180 \times 4 + b$$

$$360 = 720 + b$$

$$-360 = b.$$ 

Therefore, $y = 180x - 360$ is the equation.

Once the equation has been found, it may be used to predict or check polygon angle sums (what if $n = 100$?) that may or may not be found on the spreadsheet. Some extensions to this problem include finding the measure of each interior angle of a regular polygon. Students may want to further their investigations by finding these values using their spreadsheet. They may also want to look at concave polygons to see if their conjectures still hold or if other generalizations can be made for this different case.

Technology is very useful in the study of this problem as students are able to engage in problem solving and discovery of mathematical concepts in a short period of time. It allows students to see multiple representations including graphs, spreadsheets, and equations that allow them to make their own generalizations. Students may then continue to use the technology to confirm those generalizations. If students formulate more questions and ideas than can be answered during class, they may even leave class excited and wanting to learn more!
Conclusions

IN SUMMARY, TECHNOLOGY IS NOT just a simple tool to perform some calculations or engage in drill-and-practice exercises. Technology allows students to interact with and explore abstract and concrete concepts through multiple representations, which will enable them to be better problem solvers. Through the use of multiple computer-based applications, students can develop richer understandings of the mathematics that they encounter. When used in ways that allow students to engage in mathematical problem solving, technology becomes an important element in quality mathematics education, as described in NCTM's Principles and Standards.

Reference