Using FluidMath to Explore Recursive and Explicit Functions

FluidMath™ (www.fluiditysoftware.com), a new mathematics software tool for Tablet devices, computers, and interactive whiteboards, can create a dynamic graph or table with a simple gesture and recognize written expressions as the mathematical relationship they intend. The software uses a stylus as its input device. By changing constant values in an equation to parameters, the user can create sliders instantly and see graphs and tables change dynamically. The CAS (Computer Algebra System) functionality allows simplification of algebraic expressions and solution of equations and can perform all the calculations from algebra through calculus. FluidMath uses standard mathematical notation to explore explicitly and implicitly defined functions, parametric functions, polar functions, and recursively defined functions.

Here we will first use this software to help students investigate recursive equations, which are used as a tool to define a process that we want to carry out over and over again.

Suppose that Andrew has a headache and takes 400 mg of ibuprofen. His body eliminates 67% of the drug every 4 hours.

We write a set of recursive equations that represents the amount of the ibuprofen in Andrew’s system after several 4-hour time periods. We let $D(0)$ represent the initial amount of ibuprofen in Andrew’s system and $D(n)$ represent the amount of ibuprofen in his system after $n$ 4-hour periods. The amount of drug in his system at any 4-hour period is equal to the previous amount of drug minus what the body eliminates in that 4-hour period, giving the following equations:

$$D(0) = 400$$
$$D(n) = D(n - 1) - 0.67D(n - 1)$$

As students write the recursive system of equations, FluidMath “recognizes” their handwriting and produces the mathematics in standard print form just below the handwriting (see fig. 1).

Because we have defined an expression recursively, the software can now...
evaluate the function for a particular 4-hour period. For example, we can calculate the amount of drug in Andrew’s system after 8 hours, or two 4-hour periods (see Fig. 2). The double-arrow symbol indicates that we want the approximate value of the expression that precedes it. In this case, we can see that the amount of drug in Andrew’s system is around 43.56 mg.

We can also produce a graph of the discrete values produced by the recursive set of equations. To do so, we make what is known as a “graphing gesture” through the set of equations (see Fig. 3).

To adjust the graphing window, simply tap on the top arrow on the y-axis and write in a value such as 500 to change the maximum value. Hovering over the origin until crosshairs appear and dragging diagonally away from the origin will also change the graphing window dynamically (see Fig. 4).

Clicking on the checkbox by the Table tab at the top of the graphing screen will produce a list of numerical values for the recursive function. As shown in Figure 5, we can display all three representations for this problem simultaneously: algebraic, graphical, and numerical.

The next step in considering this set of recursive equations might be to write a closed-form, or explicit, function representing the problem. The Draw feature of FluidMath allows us to write in the file without having the software interpret the writing mathematically, almost like a comment feature. Colors available in the Draw menu box at the top left of the screen allow teachers to communicate the development of the explicit function more effectively. Figures 6 and 7, respectively, show the Draw menu and...
The software’s dynamic interface can also be used to help students understand transformations of functions. Consider the general function

\[ f(x) = p \sin(q(x + r)) + s. \]

We can explore the effects of each of the parameters \( p, q, r, \) and \( s \) on the graph of the parent function \( y = \sin x \) (see fig. 10).

Let’s define the function \( f \) as shown in figure 10 and set the values of \( r, s, \) and \( p \). The default value for each of these parameters is 1. Then we create a graph of \( f \) using the graphing gesture and add the graph of \( y = \sin x \) to the existing graph.

Now we can create a slider for \( q \) by using the slider gesture. This is a vertical mark that begins on the equals sign and ends outside the definition box for \( q \).

The numerical values for the different forms can be compared by using the Table feature. As before, click on the checkbox to display the values in a table. In figure 9, we can see that the discrete values generated by the recursive function match those of the continuous function.

Students can see from this example that FluidMath offers a way to demonstrate the connection between the discrete-valued, recursive equations and the closed-form, continuous function.

That is, for integer values of \( t \), the values of \( A(t) \) match those of the recursive function \( D(n) \). Students will make the connection that repeated multiplication by the same factor creates a geometric sequence of values that can be modeled by an exponential function.
Dragging the blue triangular widget on the slider will change the graph dynamically (see fig. 11).

FluidMath also graphs implicit, polar, and parametric functions. Figure 12 shows a few examples of these capabilities.

Another advantage of using FluidMath is the ability to create mathematical equations in standard print format. After handwriting an expression in FluidMath, we can hover over the dot in the top left corner of the expression, as shown in figure 13. Click on Copy to Clipboard as... and then choose MathML. We can easily paste the typeset mathematics expression into a Word document. This feature is a timesaver when creating examples for a handout or problems for assessments.

**FLUIDMATH AND THE COMMON CORE STANDARDS**

As these examples show, FluidMath is a powerful tool that can help teachers and students explore the behavior of functions and make sense of mathematical concepts across a variety of topics. The Common Core State Standards for Mathematics (CCSSI 2010) states that students should be able to use appropriate tools strategically as follows: “When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data…. They are able to use technological tools to explore and deepen their understanding of concepts” (p. 7). FluidMath is an example of how students and teachers can get right to the heart of mathematical discovery without the burden of learning complicated commands or accessing multiple menus.

**REFERENCE**


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